

## WEB APPENDIX CHAPTER 6

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### Nonparametric tests

In Chapter 6 we discussed parametric tests, which assume that variables tested are normally distributed. Such parametric tests are, generally, quite robust when the tested variables are not normally distributed. However, there are cases where tested variables are clearly not normally distributed. For example, a variable which indicates whether customers booked a flight in the last month can only take on the values 0 (no) or 1 (yes). The distribution of such a variable is binary and no amount of sample size will result in a test variable that can be assumed to be normally distributed. Also, continuous variables are in many cases not normally distributed. For example, distributions of household or individual income tend to have a long tail to the right while household spending tends to accelerate as income goes up.

To deal with potential problems, such as inaccurate p-values calculated, we apply nonparametric tests. Nonparametric tests do not assume a specific distribution and can be used on variables measured on a nominal or ordinal scale. A useful test for nominal or ordinal variables is the  $\chi^2$  test (pronounced as *chi square*). There are two types of  $\chi^2$ -tests involving nominal or ordinal data:

- 1.) The *one-sample  $\chi^2$ -test* (also called  $\chi^2$  goodness-of-fit test) can be used to test occurrences in a single variable's categories and compares those against expected occurrences.
- 2.) The  *$\chi^2$ -test for independence* (also known as the Pearson Chi-square test) is used to determine whether two nominal (or ordinal) variables are related.

Let's first discuss the one-sample  $\chi^2$ -test. Suppose a mobile phone producer plans to launch a new smartphone on the market but is not sure which color to use. Overall, 150 people are selected from the target group, of which 40 indicate "black," 66 "silver," and 44 "red" as their preferred smartphone color. Using a one-sample  $\chi^2$ -test we can establish whether these color preferences differ significantly from what could be expected *a priori*, by examining the differences between the observed and expected values. These a priori values can be anything a researcher tests against, such as previously established ideas or simply whether these preferences differ significantly from random assignment (150/3 groups or 50 per group). For this situation, the one-sample  $\chi^2$ -test is calculated using the following formula:

$$\chi^2 = \sum_{i=1}^k \frac{(h_i - \tilde{h}_i)^2}{\tilde{h}_i},$$

where  $h_i$  is the observed value for category  $i$  and  $\tilde{h}_i$  is the expected value for the  $i^{\text{th}}$  category, and  $k$  is the total number of categories. In other words, the  $\chi^2$ -test statistic is the sum of the squared differences between the observed and expected values, divided by the expected values.

If each category is assumed to have the same expected frequency ( $150/3=50$ ), the  $\chi^2$ -test yields the following test value:

$$\chi^2 = \frac{(40 - 50)^2}{50} + \frac{(66 - 50)^2}{50} + \frac{(44 - 50)^2}{50} = 2 + 5.12 + 0.72 = 7.84$$

The test value is not directly interpretable but must be compared to the critical value obtained from the  $\chi^2$ -statistic with  $k-1$  (in our example  $3 - 1 = 2$ ) degrees of freedom (see Table A3 in the [Web Appendix \(→Chapters→Additional Material\)](#)). As the test statistic value (7.84) is higher than the critical value (5.991;  $\alpha=0.05$ ), we can reject the null hypothesis and conclude that the observed preferences differ significantly from what could be expected. Note that in this example, we assumed that each category has the same expected frequency (i.e. 50). However, we could similarly pre-specify other expected frequencies (e.g. the takeup of the color black is 50% and the two other colors account for 25% each) and examine their differences with the observed values. Such calculations can be useful for determining production, for example.

Let's now turn to the second  $\chi^2$ -test or the  $\chi^2$ -test for independence. In the previous example, we considered only one variable, but researchers are frequently interested in evaluating whether there is a significant relationship between two nominal variables. Suppose that we further differentiated the survey described above by distinguishing between male and female respondents. A possible crosstab is presented in Table A6.1 (please ignore the  $\tilde{h}$  values in the table for the time being):

	Male	Female	$\Sigma$
Black	28 $\tilde{h}_{11} = 20$	12 $\tilde{h}_{12} = 20$	40
Silver	48 $\tilde{h}_{21} = 42$	36 $\tilde{h}_{22} = 42$	84
White	24 $\tilde{h}_{31} = 38$	52 $\tilde{h}_{32} = 38$	76
$\Sigma$	100	100	200

Table A6.1: Crosstab for  $\chi^2$ -test for independence

This 3x2 crosstab indicates that 28 male respondents prefer the black smartphone, 48 the silver smartphone, and 24 the white smartphone. The last column (row) indicates the column (row) total indicated by the summation signs ( $\Sigma$ ). In this sample, there are 100 males and 100 females. To answer the research question whether there is a relationship between the respondents' gender and their color preferences, we can apply the  $\chi^2$ -test for independence. Specifically, it tests the following hypotheses:

H<sub>0</sub>: Preference for color is independent of gender

H<sub>1</sub>: Preference for color is dependent of gender

As in the one-sample case, this test examines the degree to which the observed frequencies deviate from the expected frequencies. The expected frequency of a cell  $\tilde{h}_{ij}$  = (the column total \* the row total) / the total number of observations. Note that  $i$  refers to the index of the first variable with  $k$  categories and  $j$  refers to the index of the second variable with  $m$  categories.

This seems complicated but is not. For example, the expected frequency of the cell male/black  $\tilde{h}_{11}$  is 100 (column total category “male”) times 40 (row total of the category “black”), divided by 200 (the total number of observations), equals 20. Similarly, the expected frequency of the cell silver/male is  $\tilde{h}_{21} = \frac{100 \cdot 84}{200} = 42$ , and so on (see Table A6.1 for all cells’ expected frequencies).

The computation of the  $\chi^2$ -test statistic is similar to the example above, with the only exception that we have to append a second summation sign as there are now two nominal variables:

$$\chi^2 = \sum_{i=1}^k \sum_{j=1}^m \frac{(h_{ij} - \tilde{h}_{ij})^2}{\tilde{h}_{ij}} = \frac{(28-20)^2}{20} + \frac{(12-20)^2}{20} + \dots + \frac{(52-38)^2}{38} = 18.43$$

The degrees of freedom are calculated as follows:  $df = (k-1) \cdot (m-1)$ . This example has 2 degrees of freedom and the associated critical value (for  $\alpha = 0.05$ ) of 5.991 is much smaller than the test value (18.43). Thus, we can assume that there is a significant relationship between the respondents’ gender and preference for color.

There are several statistical measures that provide us with information regarding the strength of the association between nominal variables. Their computation only makes sense if the  $\chi^2$ -test produces significant results.

- Fisher’s exact test is a non-parametric test used only for 2 x 2 contingency table when the smallest expected value is <5. In a 2 x 2 table, the test assesses the probability of cell frequency given the marginal frequencies based on the hypergeometric probability distribution (McHugh, 2013).
- The  $\phi$  (*phi*) coefficient is used to measure the strength of association in 2x2 crosstabs. It is only a correlation coefficient for nominal variables and is computed as follows:

$$\phi = \sqrt{\frac{\chi^2}{n}}$$

If there is no association whatsoever (which is extremely unrealistic) between the variables,  $\phi$  would be zero. Conversely, a value of 1 implies that the variables are perfectly associated. Generally, a  $\phi$  value below 0.30 describes a weak association, between 0.30 and 0.49 a moderate one, and above 0.50 a strong association.

- *Kruskal Wallis* test is a non-parametric test based on rank correlation that captures how closely two pairs of data points are matched when both variables are measured at an ordinal level. When two data points are similar in the orderings of the data, the sorted list of paired observations will be close to +1 and -1 if otherwise.
- *Cramer’s V* is a modified version of the  $\phi$  (*phi*) coefficient and is used in crosstabs larger than 2x2:

$$V = \sqrt{\frac{\chi^2}{n \cdot (r - 1)}} = \sqrt{\frac{18.43}{200 \cdot (2 - 1)}} = 0.30$$

Other measures include the *likelihood-ratio chi-squared* (used when the data set is too small to meet the sample size assumption of the chi-square test that 80% of the cells have expected values of 5 or more), and *Tshuprow's T*. While all these measures are used to measure the strength of association between nominal variables, Stata provides different statistics such as *Kendall's tau-b* (pronounced as *tau*) for ordinal variables. Essentially, these measures use information on the ordering of variables' categories by considering every possible pair of cases in the crosstab. They vary between -1 and +1 and thus distinguish between positive and negative relationships. Higher absolute values denote a stronger degree of association. For more information, see, for example, Fleiss et al. (2003).

Box 6.A2: Measures of the strength of association

While the  $\chi^2$ -test for independence (as well as Fisher's exact test) helps us explore the relationship between two nominal variables from independent samples, the *McNemar test* allows us to do this when the data stem from two paired samples. More specifically, the McNemar test is used for dichotomous variables. For example, we might carry out an experiment in which we ask respondents whether they would buy a specific smartphone before and after being exposed to an online banner. The test's null hypothesis is that the number of respondents who changed their response in one direction (i.e. buying instead of not buying) is equal to the number of those who changed in the opposite direction (i.e. not buy instead of buy). The McNemar test compares the observed data to the null expectation, using a goodness-of-fit test and is interpreted like the tests discussed before. More on McNemar tests in Stata can be found here: <https://stats.idre.ucla.edu/stata/whatstat/what-statistical-analysis-should-i-usestatistical-analyses-using-stata/>.

So far, we looked at tests related to variables that are (at least) nominally scaled, but there are also various tests that are related to ordinal data. We will only look at these tests briefly, as their computation is often somewhat complex and goes beyond the scope of this book.

An important (nonparametric) test for normality is the *one-sample Kolmogorov–Smirnov (KS) test*. We can use it to test whether or not a variable is normally distributed. Technically, when assuming a normal distribution, the KS test compares the sample scores with an artificial set of normally distributed scores that has the same mean and standard deviation as the sample data. However, this approach is known to yield biased results, which are modified using the Lilliefors correction (1967). The Lilliefors correction takes into consideration that we do not know the true mean and standard deviation of the population. An issue with the KS test with the Lilliefors correction is that it is very sensitive when used on large samples and often rejects the null hypothesis if very small deviations are present. This also holds for Stata's version of the KS test, which only works well for very large sample sizes (i.e., at least 10,000 observations). Consequently, Stata does not recommend the use of a one-sample KS test (for more, read the information in Stata's help file on the KS test: <https://www.stata.com/manuals14/rksmirnov.pdf>).

The *Shapiro-Wilk* test also tests the null hypothesis that the test variable under consideration is normally distributed. Thus, rejecting the Shapiro-Wilk test provides evidence that the variable

is not normally distributed. It is best used for sample sizes of less than 50. A drawback of the Shapiro-Wilk test however, is that it works poorly if the variable you are testing has many identical values, in which case you should use the Kolmogorov-Smirnov test with Lilliefors correction.

The *Mann-Whitney U-test* is a nonparametric alternative to the independent samples *t*-test, which can be used if the dependent variable is measured on an ordinal scale. Furthermore, it is commonly applied in situations where the dependent variable is measured on an interval scale but does not follow a normal distribution. Like the *t*-test, it tests the null hypothesis that the difference in the location of two populations (expressed by the median) is zero. Rather than being based on means, the Mann-Whitney U-test statistic is based on a comparison of the observations' *ranks*. Such ranks are the number an observation gets when the observations are reorganized from the lowest (1) to the highest (x) value.

A corresponding method for paired samples exists that is called the *Wilcoxon signed-rank test*, which tests the null hypothesis that two medians stemming from paired samples are identical.

### **References for this Web Appendix**

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