WEB APPENDIX
Sarstedt, M. & Mooi, E. (2019). *A concise guide to market research. The process, data, and methods using SPSS* (3rd ed.). Heidelberg: Springer.

Two-Way ANOVA

A logical extension of a one-way ANOVA is to add a second factor variable to the analysis. For example, we could assume that, in addition to the different promotion campaigns, the management also varied the type of service provided by offering either self-service or personal service (see column "Service type" in Table 6.1). A two-way ANOVA can test for differences when we have a second factor variable. In principle, a two-way ANOVA works like a one-way ANOVA, except that the inclusion of a second factor variable necessitates the consideration of additional types of variation. Specifically, we now have to account for two types of between-group variations:

1. The between-group variation in factor variable 1 (i.e., promotion campaigns), and
2. The between-group variation in factor variable 2 (i.e., service type).

In its simplest form, the two-way ANOVA assumes that these factor variables are unrelated. However, in market research this is rarely the case, thereby requiring us to use the more complex case of related factors. When we take two related factors into account, we not only have to consider each factor variable's direct effect (also called the *main effect*) on the dependent variable, but also the factor variables' *interaction effect*. Conceptually, an interaction effect is the additional effect due to combining two (or more) factor variables. As we know from cocktail drinks, music, and paintings, there are many examples in everyday life where the whole is more than simply the sum of the parts.

In our example, the free tasting stand might be the best promotion campaign when studied separately, but it could well be that, when combined with personal service, the point of sale display is much more effective. A significant interaction effect indicates that the combination of the two factor variables is particularly effective or, on the other hand, ineffective, depending on the direction of the interaction effect. Conversely, an insignificant interaction effect suggests that we should choose the best level of the two factor variables and then use them in combination.

When running a two-way ANOVA, we have to examine the procedure's formal assumptions, which are identical to those of its one-way counterpart. While most assumptions are easily testable, SPSS does not, unfortunately, provide us with an alternative test—such as Welch's test—for situations in which we have to reject Levene's test of the homogeneity of variance. If Levene's test suggests unequal population variances, you can still interpret the analysis results (i.e., the different F-tests for the main and interaction effects), but it is better to assume a more stringent significance level, such as 0.01. Subsequently, you should consider the main and interaction effects, which are only significant if the p-values are smaller than 0.01 (instead of the commonly used threshold value of 0.05).

As discussed in Chapter 6 of the book, we need to consider various different types of variations when running a two-way ANOVA: (1) the total variation (SS_T), (2) the between-group variation in factor variable 1 (i.e. promotion campaigns; SS_{B1}), and (3) the between-group variation in factor variable 2 (i.e. service type; SS_{B2}), (4) the variation due to the interaction of factor variables 1 and 2 (SS_{B1x2}), and (5) the within-group variation (SS_W).

Given these different types of variations, there are three null hypotheses that are of particular interest to us:

1.) The factor-level means of the first factor variable are equal in the population,

2.) The factor-level means of the second factor variable are equal in the population, and

3.) There is no interaction effect between the two-factor variables.

Note that in theory, we could test further hypotheses (e.g., the population means of the first *or* second factor variable are equal), which, however, have little practical use. Before testing any of these hypotheses, we have to make sure that the assumptions described above hold. That is, we have to test whether the data are normally distributed, and whether the population variances are equal. Furthermore, this step involves ensuring that the samples are independent and that the sample sizes are similar in each group. Once this is done, we can proceed by decomposing the total variation. As you will see, we are very fortunate that SPSS does this for us, since the numerous indices can be quite confusing. Nevertheless, it's worthwhile taking a look at the formulas. You will see that the basic concept underlying the two-way ANOVA is essentially the same as with the one-way ANOVA.

Let's take a closer look at the different types of variation that need to be computed. We have already calculated the overall variation (SS_T=584), as well as the between-group variation for the first factor variable (SS_{B1}=273.80), which means we still need to consider the following three types of variation:

1.) The *between-group variation* in factor variable 2 (i.e., service type), computed by comparing each group's mean sales $\bar{x}_1$ (i.e., the sales means of the personal and self-service) with the overall mean $\bar{x}$, weighted by n_l, the number of observations in the group (i.e., 15 in both groups). Overall, there are m factor levels (groups) in factor variable 2. For the comparison, we first need to calculate the mean values of the two service types, personal service ($\bar{x}_1 = 49.27$) and self-service ($\bar{x}_2 = 46.73$). We can therefore compute the following:

$$SS_{B2} = \sum_{l=1}^{m} n_l(\bar{x}_l - \bar{x})^2 = 15 \cdot [(49.2\bar{6} - 48)^2 + (46.7\bar{3} - 48)^2] = 48.13$$

2.) To calculate the *variation due to the interaction* between the two factor variables, we need to consider yet another mean value $\bar{x}_{jl}$, which describes the mean sales in each of the six factor-level combinations (i.e., the mean sales of the personal service combined with the point of sale display ($\bar{x}_{11} = 48$) all the way through the mean sales of the self-service combined with in-store announcements ($\bar{x}_{33} = 48.30$)). n_{jl}

describes the number of observations in each of the factor level combinations, which, in our example, is always 5.

$SS_{B1x2} = \sum_{j=1}^{k}\sum_{l=1}^{m} n_{jl} \cdot (\bar{x}_{jl} - \bar{x}_{j} - \bar{x}_{l} + \bar{x}) = 5 \cdot (48 - 47.3 - 49.27 + 48) + 5 \cdot (46.60 - 47.3 - 46.73 + 48) + 5 \cdot (54.20 - 52 - 49.27 + 48) + 5 \cdot (49.80 - 52 - 46.73 + 48) + 5 \cdot (45.60 - 44.7 - 49.27 + 48) + 5 \cdot (43.80 - 44.7 - 46.73 + 48) = 13.27$

3.) The *within-group variation* SS_W. Since we have a second factor variable in our model, the overall within-group variation changes compared to that of the one-way ANOVA. Specifically, it is computed as follows:

$SS_W = \sum_{j=1}^{k}\sum_{l=1}^{m}\sum_{i=1}^{n}(x_{ijl} - \bar{x}_{jl})^2 = (50 - 48)^2 + (52 - 48)^2 + \cdots + (47 - 43.80)^2 + (42 - 43.80)^2 = 248.80$

As in the one-way ANOVA, we convert these estimates into mean squares by dividing each by its degrees of freedom, which yields the following:

– Mean square between-group variation factor variable 1 (promotion campaign):

$MS_{B1} = \frac{SS_{B1}}{k-1} = \frac{273.80}{3-1} = 136.90$

– Mean square between-group variation factor variable 2 (service type):

$MS_{B2} = \frac{SS_{B2}}{m-1} = \frac{48.13}{2-1} = 48.13$

– Mean square interaction effect: $MS_{B1x2} = \frac{SS_{B1x2}}{(k-1)\cdot(m-1)} = \frac{13.27}{2\cdot 1} = 6.63$

– Mean square within-group variation: $MS_W = \frac{SS_W}{n-k\cdot m} = \frac{248.80}{30-6} = 10.37$

We can now use these estimates to test the different effects described above. As in a one-way ANOVA, we divide the mean square values to test the desired effects: To test whether the main effect of factor variable 1 is significant, we calculate

$F = \frac{MS_{B1}}{MS_W} = \frac{163.90}{10.37} = 13.21$, which follows an F-distribution with (*k-1*) and (*n-k·m*) degrees of freedom (i.e. 2 and 24 in our example). Since the resulting critical value of 3.40 (for a significance level of 5%) is clearly smaller than the test value of 13.21, we can reject the null hypothesis and conclude that the promotion campaign factor exerts a significant influence on sales.

For the second factor variable, we obtain

$F = \frac{MS_{B2}}{MS_W} = \frac{48.13}{10.37} = 4.64$, which also follows an F-distribution, but with (*m-1*) and (*n-k·m*) degrees of freedom. As in the case of factor variable 1, the critical value (4.26 for α=0.05) is smaller than the test statistic, providing support for this type of service also having a significant bearing on sales.

Finally, we test the interaction effect by computing the following test statistic:

$F = \frac{MS_{B1x2}}{MS_W} = \frac{6.63}{10.37} = 0.64$, which follows an F-distribution, but with $(k\text{-}1)\cdot(m\text{-}1)$ and $(n\text{-}k\cdot m)$ degrees of freedom. Whereas the two main effects of factors 1 and 2 were significant, this is clearly not the case with the interaction effect. The test value (0.64) lies far below the critical value (3.40 for α=0.05).

Fig. A6.1 plots the results. The shape of the lines shows that, regardless of the type of service, the sales in both cases are highest for the free tasting stand, followed by the point of sale display, and, finally, by the in-store announcements. However, the factor variables only interact if the effect of one of the factors differs depending on the level of the other factor variable. The fact that the lines are almost parallel across the three levels of factor variable 1 shows that there is no interaction between the two factor variables. If this were the case, there would be clear differences in the lines' slopes (e.g., one line having a clear negative slope and the other having a clear positive slope).

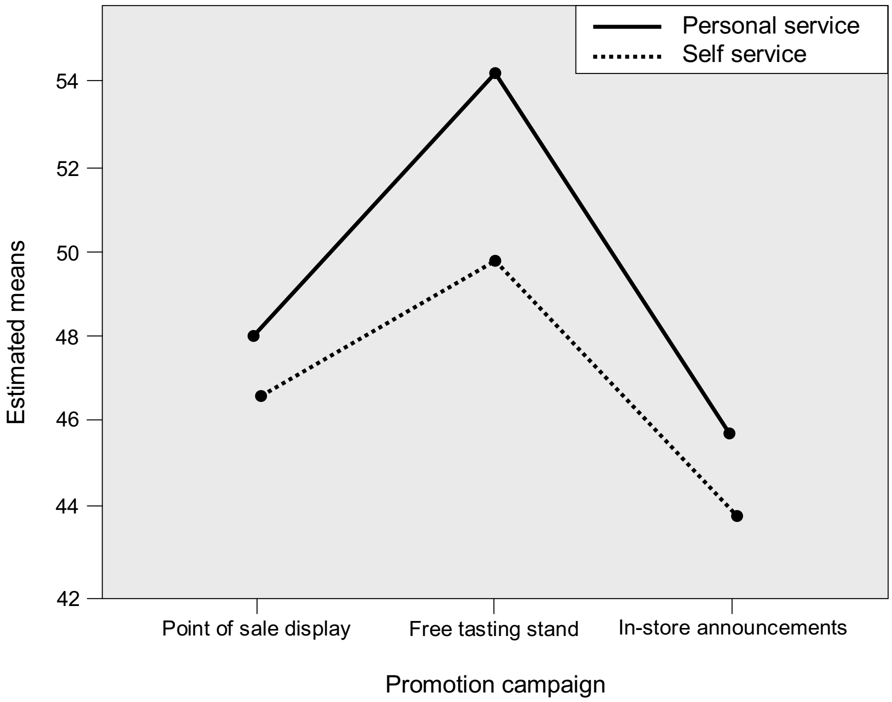

Fig. A6.1 Visual inspection of interaction effects